BENFORD'S LAW AND APPLICATIONS

This article provides an overview of Benford's Law, its many applications, and specifically how it can be used by the internal auditor.

FOR THE INTERNAL AUDITOR

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Fraud and error detection in accounting dates back to the 13th century when Europe, a monetary economy, instituted the practice of bookkeeping. Luca Pacioli, as a pioneer, built on these foundations toward developing the double-entry bookkeeping system late in the 14th century, advancing the accuracy of transaction recording. Since that time, numerous continuous improvements have materialized. However, despite these improvements, recurring problems continue to appear across a diverse set of cases.[1]

According to Jim Powell, fraudulent schemes date back to the time of Philip Ford in the 1700s. At that time, Ford embezzled large sums of money from the William Penn estates through transference of funds by way of deed. This overarching methodology continues today and has contributed to increasing instances of accounting fraud throughout the years.[2] Simultaneously, numerous methods have been created to discover and diagnose fraud, with an aim of decreasing negative economic impact and has resulted in the creation forensic accounting. Internal auditing, as a sub-genre of forensic accounting, relies on systematic approaches to evaluate and understand a company's financial statements and corresponding data. It can also be used to "improve the effectiveness of risk management, control, and governance processes."[3] Applying these procedures to a company's finan-

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cial information provides an assurance for investors and stakeholders that the information is accurate and abides to the SEC's regulations.

The role of an internal auditor is summarized by Crowder, who states that:

> When financial relationships and ratios are not consistent, there is always an internal or external reason behind the deviation. The internal auditor's job is to determine whether or not fraud is the reason for the deviation or if some other force is affecting the ratios....Such analytical review procedures are especially useful in detecting fraud in the purchasing, payroll, and revenue areas.[4]

One particular methodology, and the topic of this article, that can be mobilized to interpret these deviations, is Benford's Law, which relies on the underlying use of the digit distribution across figures in financial statements. Benford's Law is robust, in that it can be applied to many areas of internal auditing, including:

- insurance claims;
- corporate income tax;
- employee expense reports;
- vendor invoices;
- accounts receivable;
- accounts payable; and
- fixed asset records.[5]

The use of Benford's Law. while robust, also has limitations. For example, Singleton notes that:

> The IT auditor should be careful in extracting a sample and then using Benford's Law on the sample. This is especially true for directed samples in which the amount is part of the factor allowing a transaction to be chosen as the sample is not truly a random sample....The IT auditor needs to remember to make sure that the constraints (mathematical assumptions of the theory) are compatible with the data set to be tested.[6]
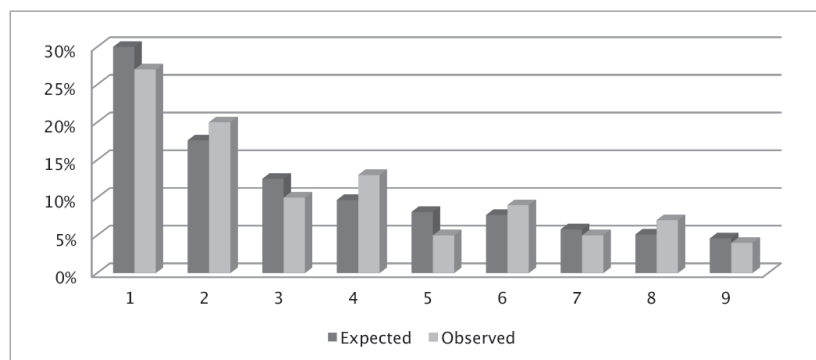
As noted, Benford's Law relies on the digit distribution found in random numbers. Specifically, Benford analyzed the digit patterns of 20 random data sets with a total of 20,229 records. His results showed that 30.6 percent of the numbers had a 1 as the first digit, 18.5 percent of the numbers had a 2 as the first digit, with continuously decreasing percentile occurrences for increasingly large digits.

Table 1 displays the expected frequencies these digits. Benford subsequently utilized a logarithmic basis as a foundation for establishing a "significant digit law" adapted from Hill. The law can be applied to second and third digits as well.

Durtschi, Hillison, and Pacini note that "Benford's analysis tests for fraudulent transcations based on whether digits appear in certain places in numbers in the expected proportion."[7]

But Nigrini further notes that "Benford's Law is quite counterintuitive; people do not naturally assume that some digits

EXHIBIT 2 Chi-Squared Test Used with Benford's Law

Adapted from: http://thebernoullitrial.wordpress.com/2010/02/23/benford-analysis-on-chinese-population-data/.

occur more frequently."[8] One possible instance of this may be seen in *United States of America vs. Charlene Corley*, where the Department of Defense, if it employed Benford's Law, may have detected an excessive occurrence of the digit 9, as the test is designed to detect data errors.[9]

How to use Benfords Law

Benford's Law may be applied directly utilizing an Excel worksheet or proprietary software programs. Numerous how-to videos and links also exist online and include adopting this methodology to large, random data sets.

Benford's Law also mobilizes a number of different statistical tests such as the chi-squared test. Simkin demonstrates this procedure and describes the chi-squared test as "a statistical test that measures how well the data distribution from a sample matches a hypothetical distribution dictated by theory."[10] The relevant test is usually referred to as a "goodness of fit" since it shows how close the expected date distribution is to an actual distribution. This test is an efficient enactor when applied to Benford's Law since it could help assess risk for the differences of distributions expressed from the data's results. This test also shows a physical representation of the results while being able to create barriers on the ends of the curve to see the amount of probability of risk of fraud there may

be with the given data that is being compared with Benford's Law. Exhibit 2 provides a snapshot of how expected versus actual frequencies might appear as a visual representation.

The robustness of Benford's Law, and its applicability to diverse portfolio of data streams, has been supported in the literature. For example, Nigrini provides a significant portfolio of activities and data records where Benford's Law is applicable. These include the interest digit frequencies from 91,022 tax returns for 1985 and 78,640 tax returns from 1988, the dollar amounts of 30,084 invoices approved for payment by an NYSE-listed oil company, and 36,515 invoices approved for payments by a software company.[11] In 2005, Nigrini analyzed revenues from 4,792 quarterly earnings in 2001 and 4,196 quarterly earnings in 2002.[12] These papers, among others, support that as a general rule, financial data within and across companies conforms reasonably well to Benford's Law.

Internal auditing provides a landscape of situations and processes that can benefit from the use of Benford's Law. Fisher stated that "Benford's Law has been used in a large number of forensic applications, including voter fraud, Greece's effort to hide its debt, and determining whether digital photographs have been altered."[13] For example, the 1990's European Union pressure to conform toward stabilizing their economy affected macro-

EXHIBIT 3 Sales to Customers

Sales Revenue to Customers X (In $)

Case 1	Case 2
1-1,014	1,844
2- 9,944	1,082
3- 8,722	2,817
4- 7,211	4,431
5- 6,301	6,742
6- 8,414	9,617
7- 9,212	1,462
8- 7,814	2,304
9- 4,423	3,273
10-3,811	3,920

Distribution by first unit

Case	1	2	3	4	5	6	7	8	9	Total
1	10%	0%	10%	10%	0%	10%	20%	20%	20%	100%
2	30%	20%	20%	10%	0%	10%	0%	0%	10%	100%

economic data. Resulting analysis suggested that Greece was noticed for significant deviations from Benford's Law. "Greek macroeconomic data is further from the Benford distribution than that of any other EU member state. Perhaps Benford analysis could have kept Greece out of the Eurozone, although it seems likely that politics would have won out over statistics."[14] This is only a handful of the situations where Benford's Law has been used to detect fraud. Browne describes Benford's Law as a "tool for pointing suspicion at frauds, embezzlers, tax evaders, sloppy accountants and even computer bugs."[15]

Illustrative examples

Provided next are two illustrative examples demonstrating the foundations of Benford's Law.

Example one

Evidence of fraudulent activities? Provided in Exhibit 3 are sales to customers from two separate business entities. Using Benford's Law, we will evaluate the presence of possible fraudulent activ-

ities or, in this case, the possibility of false sales reports.

Results. Benford's Law can be applied as an analytical review procedure (ARP) and can add to the scope of an internal audit rather easily. In this situation, we see that in case one most of the sales are skewed to a first digit beginning with 6, 7, 8, and 9 (70 percent of the time), while only 10 percent begin with 1. This is a situation that should draw the internal auditor to the possibility of fraud and will add to inquiries and to the scope of work.

In the second case, the sales correspond closely to Benford's Law, suggesting that there is no fraud. Note, however, that Benford's Law, as well as the use of other ARPs, is a first step in the internal audit scope, and this alone does not guarantee the presence or non-presence of fraud. Follow up work by the internal auditor will be required to correctly make this determination.

Example two

Suppose that a company requires a dual signature on any check written above $1,000. Below this amount, the controller signs the checks solely. The internal auditor may want to test for evidence of fraud-

ulent checks written and cashed by the controller by testing the distribution of outstanding checks. If, or example, we find that many checks written begin with a nine, a strong possibility exists that the controller is embezzling money. This is a red-flag item for the internal audit team. As an example, the controller may be embezzling funds by writing and signing checks just below the $1,000 threshold ($999, $995, and $900 for example).

Conclusion

Benford's Law is an effective way, when coupled with other procedures, to assist in the detection and prevention of financial fraud. When coupled with its cost efficiency, it is recommended that internal auditors apply Benford's Law as part of their analytical review procedures in every engagement. This provides the internal auditor with the proportion/digital distribution of the expected first digit numbers, which can then be compared with actual results, support follow-up work in the case of a material deviation, and ultimately lead to the detection and/or prevention of fraudulent activities.

This article provides an overview of Benford's Law, its many applications, and specifically how it can be used by the internal auditor. Many examples have been used whereby Benford's Law is utilized by the internal audit team. In closing, while fraudulent activities cannot be eliminated, Benford's applications will continue to undoubtedly decrease these occurrences, resulting in a savings of financial costs to the worldwide business entities and economies. ■

NOTES

[1] Lee, G.A., The coming of age of doubly entry: The Giovanni Farolfi ledger of 1299–1300, *Accounting Historians Journal* 4, no. 2 (1977): 79–95. Available at: http://umiss.lib.olemiss.edu:82/record=b1000778.

[2] Powell, J., "William Penn, America's first great champion for liberty and peace," The Freeman. Available at: http://www.quaker.org/wmpenn.html.

[3] "About the profession," The Institute of Internal Auditors. Available at: https://na.theiia.org/about-us/about-ia/Pages/about-the-profession.aspx.

[4] Crowder, N., Fraud detection techniques, *The Internal Auditor* 54, no. 2 (1997): 17–20.

[5] "Auditor's guide to tests using Benford's Law," EZ-R Stats, LLC. Available at: http://www.ezrstats.com/doc/Auditors_Guide_to_Tests_using_Benfords_Law.pdf.

[6] Singleton, T.W., Understanding and applying Benford's Law, *ISACA Journal* 3 (2011). Available at: http://www.isaca.org/Journal/Past-Issues/2011/Volume-3/Pages/Understanding-and-Applying-Benfords-Law.aspx.

[7] Durtschi, C., Hillison, W., and Pacini, C., The effective use of Benford's Law to assist in detecting fraud in accounting data, *Journal of Forensic Accounting* 5 (2004): 17–34. Available at: http://faculty.usfsp.edu/gkearns/Articles_Fraud/Benford%20Analysis%20Article.pdf.

[8] Nigrini, M.J., I've got your number: How a mathematical phenomenon can help CPAs uncover fraud and other irregularities, Journal Of Accountancy (1999). Available at: http://www.journalofaccountancy.com/issues/1999/may/nigrini.

[9] *Ibid.*

[10] Simkin, M.G., Using spreadsheets and Benford's Law to test accounting data, *ISACA Journal* 1 (2010). Available at: http://www.isaca.org/Journal/Past-Issues/2010/Volume-1/Pages/Using-Spreadsheets-and-Benford-s-Law-to-Test-Accounting-Data1.aspx.

[11] Nigrini, M.J. and Mittermaier, L.J., "The use of Benford's Law as an aid in analytical procedures," *Auditing: A Journal of Practice and Theory* 2, no. 16 (1997): 52–67; Drake, P.D. and Nigrini, P.D., "Computer assisted analytical procedures using Benford's Law," *Journal of Accounting Education* 18 (2000): 127–146.

[12] Nigrini, M.J., "An assessment of the change in the incidence of earnings management around the Enron–Andersen episode," Review of Accounting & Finance 4, no. 1 (2005): 92–110.

[13] Fisher, D., The simple mathematical law that financial fraudsters can't beat, *Forbes* (May 30, 2011). Available at: http://www.forbes.com/sites/danielfisher/2014/05/30/the-simple-mathematical-law-that-financial-fraudsters-cant-beat/.

[14] Harford, T., Benford's Law and the eurozone crisis, Business Life (Nov 25, 2011). Available at: http://businesslife.ba.com/Ideas/Economics/would-benfords-law-have-caught-out-greece.html.

[15] Browne, M.W., Benford's Law, *New York Times* (Aug 4, 1998). Available at: http://www.edn.com/electronics-blogs/anablog/4399245/Benford-s-Law.